Name: _____

GCE Stats section 9

Contingency tables

practice questions from GCE Maths

Date:

Question parts in red cover content in the GCE Statistics specification but outside this section of the scheme of work

Time:

Total marks available:

Total marks achieved: _____

Questions

Q1.

A doctor takes a random sample of 100 patients and measures their intake of saturated fats in their food and the level of cholesterol in their blood. The results are summarised in the table below.

Intake of saturated fats \ Cholesterol level	High	Low
High	12	8
Low	26	54

Using a 5% level of significance, test whether or not there is an association between cholesterol level and intake of saturated fats. State your hypotheses and show your working clearly.

(Total 10 marks)

Q2.

A new drug to vaccinate against influenza was given to 110 randomly chosen volunteers. The volunteers were given the drug in one of 3 different concentrations, *A*, *B* and *C*, and then were monitored to see if they caught influenza. The results are shown in the table below.

	A	*B*	*C*
Influenza	12	29	9
No influenza	15	23	22

Test, at the 10% level of significance, whether or not there is an association between catching influenza and the concentration of the new drug. State your hypotheses and show your working clearly. You should state your expected frequencies to 2 decimal places.

(Total for question = 10 marks)

Q3.

A survey asked a random sample of 200 people their age and the main use of their mobile phone.

The results are shown in Table 1 below.

		Main use of their mobile phone		
		Internet	**Texts**	**Phone calls**
Age	**Under 20**	27	14	9
	From 20 to 40	32	34	29
	Over 40	15	19	21

Table 1

The data are to be used to test whether or not age and main use of their mobile phone are independent.

Table 2 shows the expected frequencies for each group, assuming people's age and main use of their mobile phone are independent.

		Main use of their mobile phone		
		Internet	**Texts**	**Phone calls**
Age	**Under 20**	18.5	16.75	14.75
	From 20 to 40	35.15	31.825	28.025
	Over 40	20.35	18.425	16.225

Table 2

(a) For users under 20 choosing the Internet as the main use of their mobile phone,

(i) verify that the expected frequency is 18.5
(ii) show that the contribution to the χ^2 test statistic is 3.91 to 3 significant figures.

(2)

(b) Given that the χ^2 test statistic for the data is 9.893 to 3 decimal places, test at the 5% level of significance whether or not age and main use of their mobile phone are independent. State your hypotheses clearly.

(5)

(Total 7 marks)

Q4.

A number of males and females were asked to rate their happiness under the headings "not happy",
"fairly happy" and "very happy".

The results are shown in the table below

		Happiness			Total
		Not happy	Fairly happy	Very happy	
Gender	Female	9	43	34	86
	Male	13	25	16	54
Total		22	68	50	140

Stating your hypotheses, test at the 5% level of significance, whether or not there is evidence of an association between happiness and gender. Show your working clearly.

(Total 10 marks)

Q5.

A factory manufactures batches of an electronic component. Each component is manufactured in one of three shifts. A component may have one of two types of defect, D_1 or D_2 , at the end of the manufacturing process. A production manager believes that the type of defect is dependent upon the shift that manufactured the component. He examines 200 randomly selected defective components and classifies them by defect type and shift. The results are shown in the table below.

Defect type / Shift	D_1	D_2
First shift	45	18
Second shift	55	20
Third shift	50	12

Stating your hypotheses, test, at the 10% level of significance, whether or not there is evidence to support the manager's belief. Show your working clearly.

(Total 10 marks)

Q6.

A psychologist carries out a survey of the perceived body weight of 150 randomly chosen people. He asks them if they think they are underweight, about right or overweight. His results are summarised in the table below.

	Underweight	About right	Overweight
Male	20	22	30
Female	16	28	34

The psychologist calculates two of the expected frequencies, to 2 decimal places, for a test of independence between perceived body weight and gender. These results are shown in the table below.

	Underweight	About right	Overweight
Male	17.28		
Female	18.72		

(a) Complete the table of expected frequencies shown above.

(2)

(b) Test, at the 10% level of significance, whether or not perceived body weight is independent of gender. State your hypotheses clearly.

(7)

The psychologist now combines the male and female data to test whether or not body weight types are chosen equally.

(c) Find the smallest significance level, from the tables in the formula booklet, for which there is evidence of a preference.

(5)

(Total for question = 14 marks)

Q7.

John thinks that a person's eye colour is related to their hair colour. He takes a random sample of
600 people and records their eye and hair colours. The results are shown in Table 1.

		Hair colour				
		Black	Brown	Red	Blonde	Total
Eye colour	Brown	45	125	15	58	243
	Blue	34	90	10	58	192
	Hazel	20	38	16	26	100
	Green	6	29	7	23	65
	Total	105	282	48	165	600

Table 1

John carries out a χ^2 test in order to test whether eye colour and hair colour are related. He calculates the expected frequencies shown in Table 2.

		Hair colour			
		Black	Brown	Red	Blonde
Eye colour	Brown	42.5	114.2	19.4	66.8
	Blue	33.6	90.2	15.4	52.8
	Hazel	17.5	47	8	27.5
	Green	11.4	30.6	5.2	17.9

Table 2

(a) Show how the value 47 in Table 2 has been calculated.

(1)

(b) Write down the number of degrees of freedom John should use in this χ^2 test.

(1)

Given that the value of the χ^2 statistic is 20.6, to 3 significant figures,

(c) find the smallest value of α for which the null hypothesis will be rejected at the α% level of significance.

(1)

(d) Use the data from Table 1 to test at the 5% level of significance whether or not the proportions of people in the population with black, brown, red and blonde hair are in the ratio 2:6:1:3

State your hypotheses clearly.

(9)

(Total 12 marks)

Q8.

Two breeds of chicken are surveyed to measure their egg yield. The results are shown in the table below.

Breed \ Egg yield	Low	Medium	High
Leghorn	22	52	26
Cornish	14	32	4

Showing each stage of your working clearly, test, at the 5% significance level, whether or not there is an association between egg yield and breed of chicken. State your hypotheses clearly.

(Total 10 marks)

Q9.

A random sample of 100 people were asked if their finances were worse, the same or better than this
time last year. The sample was split according to their annual income and the results are shown in
the table below.

Annual income \ Finances	Worse	Same	Better
Under £15 000	14	11	9
£15 000 and above	17	20	29

Test, at the 5% level of significance, whether or not the relative state of their finances is independent of their income range. State your hypotheses and show your working clearly.

(Total 10 marks)

Q10.

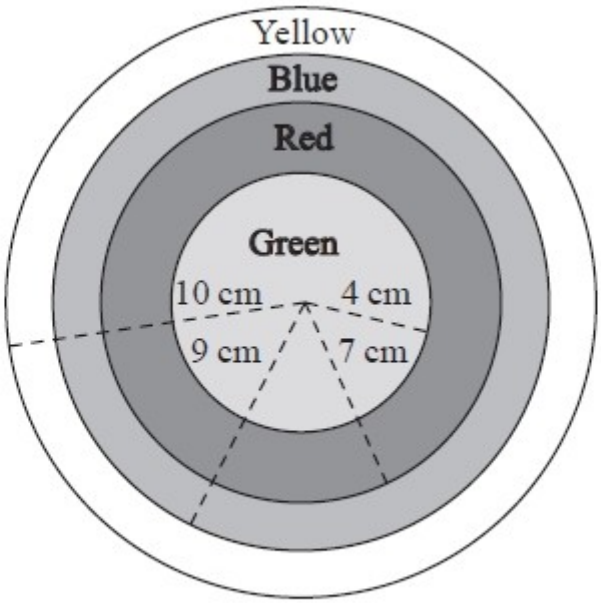

Figure 1

The sketch in Figure 1 represents a target which consists of 4 regions formed from 4 concentric circles of radii 4 cm, 7 cm, 9 cm and 10 cm. The regions are coloured as labelled in Figure 1.

A random sample of 100 children each choose a point on the target and their results are summarised in the table below.

Colour of region	Green	Red	Blue	Yellow
Frequency	22	39	25	14

Caitland is trying to model the distribution of the points chosen by the children. She defines the random variable D to be the distance, in cm, of a point from the centre of the target and assumes $D \sim U[0, 10]$.

(a) Stating your hypotheses clearly and using a 1% level of significance, test whether or not $U[0, 10]$ is a suitable model for these data.

(9)

Henry claims that the points are randomly distributed over the target and the probability of a point being in any particular region is proportional to the area of that region. He calculates expected frequencies and obtains the following table.

Colour of region	Green	Red	Blue	Yellow
Expected frequency	16	33	r	s

(b) Find the value of r and the value of s.

(3)

Henry obtained a test statistic of 6.188 and no groups were pooled.

(c) State what conclusion Henry should make about his claim.

(2)

Phoebe believes that the children chose the region of the target according to colour. She believes that boys and girls would favour different colours and splits the original data by gender to obtain the following table.

Observed frequencies

Colour of region	Green	Red	Blue	Yellow	Total
Boys	10	12	10	3	35
Girls	12	27	15	11	65

(d) State suitable hypotheses to test Phoebe's belief.

(1)

Phoebe calculated the following expected frequencies to carry out a suitable test.

Expected frequencies

Colour of region	Green	Red	Blue	Yellow
Boys	7.7	13.65	8.75	4.9
Girls	14.3	25.35	16.25	9.1

(e) Show how the value of 25.35 was obtained.

(1)

Phoebe carried out the test using 2 degrees of freedom and a 10% level of significance.
She obtained a test statistic of 1.411

(f) Explain clearly why Phoebe used 2 degrees of freedom.

(1)

(g) Stating your critical value clearly, determine whether or not these data support

Phoebe's belief.

(2)

(Total for question = 19 marks)

Examiner's Report

Q1.

This proved to be a friendly opening to the paper with very few failing to show sufficient working and many scoring full marks. Some lost marks for the hypotheses either through laziness (simply stating "no association" for the null hypothesis is not sufficient as we want to see the variables under consideration being mentioned) or for stating them the wrong way around. The calculations were usually correct but some mistakes occurred in the degrees of freedom and the conclusion was not always given in context.

Q2.

Hypotheses and final conclusions were usually stated correctly although some were imprecise, missing out either 'concentration' or 'drug'. Many students were accurate in their calculations and most had the correct degrees of freedom. A few, however, read the critical value from the wrong column in the tables. Not all students are showing the calculation needed for expected values, but the majority gave a correct explicit statement regarding their null hypothesis based on their values.

Q3.

Full marks were often awarded in question (a) as the methods were well understood and were calculated accurately. Some confusion was evident in the formulation of the hypotheses in question (b), but again full marks here was not unusual.

Q4.

This was a routine question for the majority of students. Both the hypotheses and the result of the test were usually given in context and many fully correct solutions were seen here. Premature rounding proved costly in some solutions offered and typically these students did not show all their calculations

Q5.

This was done very well by most candidates. Calculations were accurate and well set out, hypotheses clearly stated and a correct critical value was common. Some candidates used the easier calculation method and very few confused the solution with correlation.

Q6.

Part (a) was fully correct for the majority of students who were able to state their hypotheses using the correct context and key words. However, some are still incorrectly writing there is an 'association'. All but the very weakest of students were able to calculate the correct test statistic, but students should be advised to check every calculation thoroughly as incorrect answers were generally down to a misplaced digit in one of their calculations.

Part (c) was more of a challenge. Those who attempted it generally managed to get to a correct test statistic, but were then unsure how to proceed or simply stated 1% was the lowest significance level with little understanding of how to check this. If a student set off on a route with O and E the same, this method usually resulted in no marks being awarded.

Q7.

The χ^2 tests were only examined in this one question on this paper and a number of candidates failed to identify the fairly straightforward goodness of fit test in part (d). The first three parts were answered well and most scored full marks although a few misread the tables and thought α was 1 and some gave α as 0.025. In part (d) the common error was to simply calculate the test statistic for a test of association between hair colour and eye colour rather than using the column totals from Table 1 to conduct the goodness of fit test on the hair colour ratios. Those who did attempt the correct test could usually calculate the expected frequencies and often completed the test accurately.

Q8.

This was a ready source of marks to the great majority of candidates. The only common error seen was in the statement of the hypotheses and the conclusion, which were sometimes incomplete.

Q9.

For most candidates this question was a good source of marks. Hypotheses were usually correctly phrased in terms of "independence" or "association" and the calculations were usually clearly set out although some inappropriate rounding sometimes gave an answer of 3.56. The degrees of freedom and critical value caused few problems and most gave a correct conclusion in context.

Q10.

This question allowed competent students to score very well. Far too many students did the first test with an assumption of equal expected values. This probably was a result of not having read the question carefully enough. Students found part (b) and part (e) a good source of marks and part (c) only marginally less so. Too many students were unable to express their hypotheses precisely enough in part (d) but were able to successfully give the correct interpretation of the test result in part (g). Most students were able to explain the calculation of the degrees of freedom in part (f), but too many thought that the pooling of two columns would simply lead to the loss of 1 degree of freedom from the original number.

Mark Scheme

Section 9	Contingency tables
qu.1	June 2013 S3 - Q1
qu.2	May 2016 S3 - Q2
qu.3	May 2014 S3R - Q2
qu.4	May 2014 S3 - Q3
qu.5	June 2011 S3 - Q3
qu.6	May 2017 S3 - Q4
qu.7	June 2013 S3R - Q4abc
qu.8	June 2012 S3 - Q4
qu.9	June 2010 S3 - Q5
qu.10	May 2015 S3 - Q6defg

Q1.

Question Number	Scheme	Marks

Cholesterol Level	High	Low	
High	7.6	12.4	20
Low	30.4	49.6	80
	38	62	100

M1A1 (2)

H_0 : Cholesterol level is independent of intake of saturated fats(no association) B1

H_1 : Cholesterol level is not independent of intake of saturated fats (association) (1)

O	E	$\frac{(O-E)^2}{E}$	$\frac{O^2}{E}$
12	7.6	2.547… or $\frac{242}{95}$	18.947… or $\frac{360}{19}$
8	12.4	1.56129… or $\frac{242}{155}$	5.161… or $\frac{160}{31}$
26	30.4	0.6368… or $\frac{121}{190}$	22.236… or $\frac{845}{38}$
54	49.6	0.3903… or $\frac{121}{310}$	58.790… or $\frac{3645}{62}$

dM1 A1

$\sum\frac{(O-E)^2}{E}=5.1358234..$ or $\frac{1.2^2}{7.6}+\frac{8^2}{12.4}+\frac{26^2}{30.4}+\frac{54^2}{49.6}-100=5.14$ (awrt **5.14**) A1 (3)

$\nu=(2-1)(2-1)=1$ B1

$\chi^2_1(0.05)=3.841$ B1 (2)

$5.14 > 3.841$ so sufficient evidence to reject H_0 [Condone "accept H_1"] M1

Association between cholesterol level and saturated fat intake A1 (2)

Total 10

Notes

Minimum working use part marks: E_i (2), Hyp (1), 5.14 (3), 3.841 (2), Conclusion (2)

1st M1 for some use of $\frac{\text{Row Total}\times\text{Col.Total}}{\text{Grand Total}}$. May be implied by correct E_i

1st A1 for all expected frequencies correct. Allow M1A0 for E_i rounded to integers

1st B1 for both hypotheses. Must mention "cholesterol" and "fats" at least once
Use of "relationship" or "correlation" or "connection" is B0

2nd dM1 for at least 2 correct terms (as in 3rd or 4th column) or correct expressions with their E_i
Dependent on 1st M1 Accept 2sf accuracy for the M mark

2nd A1 for all correct terms. May be implied by a correct ans.(2 dp or better)
Allow truncation eg 2.54… 3rd A1 for awrt 5.14

2nd B1 for correct degrees of freedom (may be implied by a cv of 3.841)

3rd M1 for a correct statement linking their test statistic and their cv(cv could be 2.705 or > 3.5)
Contradictory statements score M0 e.g. "significant, do not reject H_0"

4th A1 for a correct comment in context - must mention "cholestorol" and "fats"
condone "relationship" or "connection" here but **not** "correlation".
e.g. "There is evidence of a relationship between cholesterol level and fat intake"
No follow through. If e.g hypotheses are the wrong way round A0 here.

Q2.

Question Number	Scheme	Marks
	H_0 : **Drug concentration** and catching **influenza** are independent / not associated	
	H_1 : **Drug concentration** and catching **influenza** are not independent / associated	B1

	A	*B*	*C*	
Influenza	$\frac{50\times27}{110}=12.272...$	$\frac{50\times52}{110}=23.636...$	$\frac{50\times31}{110}=14.090...$	50
No influenza	$\frac{60\times27}{110}=14.727...$	$\frac{60\times52}{110}=28.363...$	$\frac{60\times31}{110}=16.909...$	60
	27	52	31	110

M1A1

O	E	$\frac{(O-E)^2}{E}$	$\frac{O^2}{E}$
12	12.272…	0.0060…	11.7333…
29	23.636…	1.2171…	35.5807…
9	14.090…	1.8392…	5.7483…
15	14.727…	0.0050…	15.2777…
23	28.363…	1.0142…	18.6506…
22	16.909…	1.5327…	28.6236…

M1A1

$\sum\frac{(O-E)^2}{E}=5.6145...$ or $\sum\frac{O^2}{E}-N=115.62...-110=5.6145...$ awrt 5.61-5.62 — A1

$\nu=(3-1)(2-1)=2$, $\chi^2_2(10\%)=4.605$ — B1B1ft

Reject H_0 — M1

Drug concentration and catching **influenza** are not independent / are associated. — A1cao

(10)

Total 10

Notes

B1 hyps correct way around

M1 for correct expression at least once

A1 **all seen** and correct 2dp or better. Can be implied by test statistic of awrt 5.61-5.62.

M1 either method at least one correct

A1 at least 3 correct values. Can be implied by test statistic of awrt 5.61-5.62

A1 awrt 5.61-5.62

B1 cao

B1 follow through their ν

M1 must correctly reject / not reject the null hypothesis based on their test stat and cv oe

Q3.

Question Number	Scheme	Marks
(a) (i) (ii)	Expected value $= \frac{50\times74}{200} = 18.5$	B1 cso
	χ^2 contribution $= \frac{(27-18.5)^2}{18.5} = 3.905405405 = 3.91$ to 3sfs	B1 cso
		(2)
(b)	H_0: users age and main mobile phone use are independent/ no association between users age and main mobile phone use H_1: users age and main mobile phone use are not independent/ some association between users age and main mobile phone use	B1
	$\nu = 4$	B1
	Critical value $\chi^2 = 9.488$	B1ft
	Test statistic is in critical region therefore significant evidence to reject H_0 and accept H_1. Evidence at 5% level that age and main phone use are not independent.	M1 A1ft
		(5) (7 marks)

Notes	
(b)	3^{rd} B1 ft on their value of ν M1 for attempt to compare test statistic and their critical value A1 ft on test statistic and critical value but must be comment in context. (A0 if hypotheses are the wrong way around)

Q4.

Question Number	Scheme	Marks

		Happiness		
		Not happy	Fairly happy	Very happy
Gender	Female	13.51	41.77	30.71
	Male	8.49	26.23	19.29

M1
A1

H_0 : Happiness and gender are independent/ not associated
H_1 : Happiness and gender are not independent/ associated

B1

O	E	$\frac{(O-E)^2}{E}$	$\frac{O^2}{E}$
9	13.51	1.508	5.996
43	41.77	0.0361	44.264
34	30.71	0.351	37.637
13	8.49	2.402	19.915
25	26.23	0.0575	23.829
16	19.29	0.560	13.274

dM1

A1

$\sum \frac{(O-E)^2}{E} = 4.91$ or $\sum \frac{O^2}{E} - N = 144.91 - 140 = 4.91$

A1

$\nu = (3-2)(2-1) = 2$

B1

$\sum \frac{(O-E)^2}{E} < 5.991$

B1ft

4.91 < 5.991 so 'insufficient evidence to reject H0 ' or 'Accept H0'

M1

No association between gender and happiness.

A1

(10)
Total 10

Notes

1st M1 for some use of $\frac{Row\ Total \times Column\ Total}{Grand\ Total}$. May be implied by at least 1 correct *Ei*

1st A1 awrt 13.5, 41.8, 30.7, 8.5, 26.2 and 19.3 Allow M1A0 for *Ei* rounded to integers

1st B1 for both hypotheses. Must mention "happiness" and "gender" at least once.

Use of "relationship" or "correlation" or "connection" is B0

2nd dM1 for at least 2 correct terms (in 3rd or 4th columns) or correct expressions with their *Ei*

Dependent on 1st M1. Accept 2sf accuracy for the M mark.

2nd A1 for all correct terms (2sf or better). May be implied by a correct ans

Allow truncation e.g. 44.2…

3rd A1 awrt 4.91 . Condone 4.915

2nd B1 for correct degrees of freedom (may be implied by a cv of 5.991)

3rd B1ft for cv that follows from their degrees of freedom

3rd M1 for a correct statement linking their test statistic and their cv

Contradictory statements score M0 e.g. "significant, do not reject H0"

Condone "reject H1 "

4th A1 for a correct comment in context - must mention "gender" and "happiness"

Condone "relationship" or "connection" here but **not** "correlation".

e.g. "There is no evidence of a relationship between gender and happiness"

No follow through. If e.g hypotheses are the wrong way around A0 here.

SC Use of calculator with **no working** may get M0A0B1M1A0A1B1B1M1A1

Q5.

Question Number	Scheme	Marks

Defect Type / Shift	D_1	D_2	
First Shift	47.25	15.75	63
Second Shift	56.25	18.75	75
Third Shift	46.5	15.5	62
	150	50	200

M1A1

H_0 : Type of defect is independent of Shift (no association)

H_1 : Type of defect is not independent of Shift (association)

B1

O	E	$\frac{(O-E)^2}{E}$	$\frac{O_i^2}{E_i}$
45	47.25	0.1071…	42.857...
18	15.75	0.3214…	20.571..
55	56.25	0.02777…	53.777...
20	18.75	0.0833…	21.333...
50	46.5	0.2634…	53.763...
12	15.5	0.7903…	9.290...

M1A1

$\frac{(O-E)^2}{E}$ =1.5934.. or $\frac{O_i^2}{E_i}$ -200=201.5934-200=1.5934.. awrt1.59 A1

$\nu = (3-1)(2-1) = 2$ B1

$\chi^2_2(0.10) = 4.605$ B1ft

1.59<4.605 so insufficient evidence to reject H_0 M1

Insufficient evidence to support manager's belief /claim. A1

10

Notes

1st M1 for some use of $\frac{\text{Row Total} \times \text{Col.Total}}{\text{Grand Total}}$ May be implied by correct E_i

1st A1 for all expected frequencies correct

B1 for both hypotheses. Must mention "defect" and "shift" at least once

Use of "relationship" or "correlation" or "connection" is B0

2nd M1 for at least two correct terms (as in 3rd or 4th column) or correct expressions with their E_i

2nd A1 for all correct terms. May be implied by a correct answer.(2 dp or better-allow eg 0.10…)

3rd M1 for a correct statement linking their test statistic and their cv . Must be χ^2 not normal.

4th A1 for a correct comment in context - must mention "manager's belief" or "shift" and "defect type" - condone "relationship" or "connection" here but **not** "correlation". No follow through e.g. "There is evidence of a relationship between shift and type of defect"

Q6.

Question Number	Scheme	Notes	Marks
(a)	$\frac{72\times50}{150}=24,\ \frac{78\times50}{150}=26$	For one correct $\frac{\text{Row Total}\times\text{Column Total}}{\text{Grand Total}}$; can be implied by correct answers.	M1
	$\frac{72\times64}{150}=30.72,\ \frac{78\times64}{150}=33.28$	24, 26, 30.72, 33.28 only.	A1
			(2)
(b)	H_0 : Perceived (body) weight is independent of gender (no association) H_1 : Perceived (body) weight is not independent of gender (association)	Both hypotheses required. Must mention "Perceived", "weight" and "gender" at least once. Use of "relationship" or "correlation" or "connection" or "link" award B0.	B1
	O | E | $\frac{(O-E)^2}{E}$ | $\frac{O^2}{E}$ 20 | 17.28 | 0.428148 | 23.14815 22 | 24 | 0.166667 | 20.16667 30 | 30.72 | 0.016875 | 29.29688 16 | 18.72 | 0.395214 | 13.67521 28 | 26 | 0.153846 | 30.15385 34 | 33.28 | 0.015577 | 34.73558 150 | 150 | 1.176327 | 151.1763	M1 for at least 2 correct terms (as in 3rd or 4th column) or correct expressions. A1 for all correct. Accept 2sf accuracy. Allow truncation e.g. 1.17…	M1A1
	$\sum\frac{(O-E)^2}{E}$ or $\sum\frac{O^2}{E}-150=1.18$	Awrt 1.18-1.19	A1
	$\nu=(3-1)(2-1)=2,\ \chi^2_2(10\%)=4.605$	2 can be implied by 4.605 seen	B1B1ft
	(Accept H_0) Perceived (body) **weight** is independent of **gender** (no association)	A correct comment in context - must mention "weight" and "gender". Condone "relationship" or "connection" here but **not** "correlation". Follow through from their test stat and cv, but **hypotheses must be correct.**	A1ft
			(7)

(c)	O | E | $\frac{(O-E)^2}{E}$ | $\frac{O^2}{E}$ 36 | 50 | 3.92 | 25.92 50 | 50 | 0 | 50 64 | 50 | 3.92 | 81.92 150 | 150 | 7.84 | 157.84	B1 for $E_i=50$, could be implied. M1 for combining values and for attempting $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$ with at least 2 correct expressions or values. A1 for all correct, can be implied by correct answer below.	B1 M1A1
	$\sum\frac{(O-E)^2}{E}$ or $\sum\frac{O^2}{E}-150=7.84$	Awrt 7.84	A1
	$\nu=2,\ \chi^2_2(2.5\%)=7.378$	0.025 or 2.5%	A1
			(5)
			Total 14

Q7.

Question Number	Scheme	Marks
(a)	$\frac{282\times100}{600}$ (Do not accept 282 – 114.2 – 90.2 – 30.6 (o.e.))	B1 **(1)**
(b)	9	B1 **(1)**
(c)	2.5 or better (Do not accept 0.025)	B1 **(1)**
(d)	H_0: hair colour occurs in the ratio 2:6:1:3 H_1: hair colour does not occur in the ratio 2:6:1:3	B1

	black	brown	red	blonde
observed	105	282	48	165
expected	100	300	50	150
$\frac{(O_i-E_i)^2}{E_i}$	0.25	1.08	0.08	1.5
$\frac{O_i^2}{E_i}$	110.25	265.08	46.08	181.5

B1 expected
M1
A1

$\sum \frac{(O_i-E_i)^2}{E_i} = 2.91$ or $\sum \frac{O_I^2}{E_I} - 600 = 602.91 - 600 = 2.91$ (awrt **2.91**) A1

$\nu = 3$ B1

cv is 7.815 B1

[2.91< 7.815] so insufficient evidence to reject H_0 or not significant dM1

There is evidence to suggest that hair colour does occur in the given ratio. A1

(9)
[Total 12]

	Notes
(d)	1st B1 for both hypotheses. Must mention hair colour and ratio e.g. "hair colour in the given ratio" Allow use of ditto 2nd B1 for all 4 correct expected frequencies 1st M1 for at least 2 correct calculations from 3rd or 4th row 1st A1 for all correct calculations to at least 3sf if row 4 If awrt 2.91 is seen with no incorrect working award B1M1A1A1 2nd dM1 Dep on 1st M1 for a correct statement linking their test statistic and their cv (cv > 3.5) 3rd A1 for a correct comment in context - must mention "hair colour" and "ratios" or "model" e.g. "There is evidence of to support the given model" No follow through If hypotheses are the wrong way round score A0.

Q8.

Question Number	Scheme	Marks
	H_0 : Egg yield and breed of chicken are independent (not associated) H_1 : Egg yield and breed of chicken are dependent (associated)	B1

Egg Yield / Breed	Low	Medium	High	Total
Leghorn	$\frac{100\times36}{150}=24$	$\frac{100\times84}{150}=56$	$\frac{100\times30}{150}=20$	100
Cornish	$\frac{50\times36}{150}=12$	$\frac{50\times84}{150}=28$	$\frac{50\times30}{150}=10$	50
Total	36	84	30	150

M1A1

O	E	$\sum\frac{(O-E)^2}{E}$	$\sum\frac{O^2}{E}$
22	24	0.166667	20.2
52	56	0.285714	48.3
26	20	1.8	33.8
14	12	0.333333	16.3
32	28	0.571429	36.6
4	10	3.6	1.6

M1A1

$\sum\frac{(O-E)^2}{E}=6.757...$ or $\sum\frac{O^2}{E}-100=6.757...$ A1

$\nu=2, \chi^2_2(5\%)=5.991$ B1B1ft

(6.757>5.991 so sufficient evidence to) reject H_0 M1

Egg yield and breed of chicken are dependent (associated) A1

(10)

Total 10

Notes

B1 for both hypotheses. Must mention "yield" and "breed" in both but condone ditto marks.
Use of "relationship" or "correlation" or "connection"is B0

1st M1 for some use of $\frac{\text{Row Total}\times\text{Col.Total}}{\text{Grand Total}}$. May be implied by a correct E_i

1st A1 for all expected frequencies correct

2nd M1 for at least two correct terms or correct expressions with their E_i

2nd A1 for all correct terms. May be implied by a correct answer (2 sf or better)

3rd M1 for a correct statement linking their test statistic and their cv . Must be χ^2 not normal.

4th A1 for a correct comment in context - must mention "egg yield" and "breed of chicken" - condone "relationship" or "connection" here but not "correlation". No follow through e.g. "There is no evidence of a relationship between egg yield and breed of chicken" is A0 whatever their test stat or cv.

Q9.

Question Number	Scheme	Marks

Income \ Finances	Worse	Same	Better	
Under £15 000	10.54	10.54	12.92	34
£15 000 and above	20.46	20.46	25.08	66
	31	31	38	100

M1
A1

H_0 : State of finances and income are independent (not associated)

H_1 : State of finances and income are not independent (associated)

B1

O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$
14	10.54	1.1358….	18.59..
11	10.54	0.0200….	11.48..
9	12.92	1.1893…	6.269..
17	20.46	0.5851…	14.12..
20	20.46	0.0103…	19.55..
29	25.08	0.6126…	33.53..

M1

A1

$\sum \frac{(O_i-E_i)^2}{E_i} = 3.553\ldots$ or $\sum \frac{O_i^2}{E_i} - 100 = 103.553\ldots - 100 = 3.553\ldots$ (awrt **3.55**) A1

$\nu = (3-1)(2-1) = 2$ B1

cv is 5.991 B1

3.553 < 5.991 so insufficient evidence to reject H_0 or not significant M1

There is no evidence of association between state of finances and income. A1

10

1st M1 for some use of $\frac{\text{Row Total} \times \text{Col.Total}}{\text{Grand Total}}$. May be implied by correct E_i

1st A1 for all expected frequencies correct

B1 for both hypotheses. Must mention "state" or "finances" and "income" at least once
Use of "relationship" or "correlation" or "connection"is B0

2nd M1 for at least two correct terms (as in 3rd or 4th column) or correct expressions with their E_i

2nd A1 for all correct terms. May be implied by a correct answer.(2 dp or better-allow eg 1.13…)

3rd M1 for a correct statement linking their test statistic and their cv . Must be χ^2 not normal.

4th A1 for a correct comment in context - must mention "state" or "finances" and "income"
condone "relationship" or "connection" here but **not** "correlation". No follow through.
e.g. "There is no evidence of a relationship between finances and income"

Q10.

Question Number	Scheme	Marks
(a)	H_0 : U[0, 10] is a suitable model H_1 : U[0, 10] is not a suitable model	B1
	D | O_i | E_i | $\frac{(O_i-E_i)^2}{E_i}$ | $\frac{O_i^2}{E_i}$ 0 – 4 | 22 | 40 | 8.1 | 12.1 4 – 7 | 39 | 30 | 2.7 | 50.7 7 – 9 | 25 | 20 | 1.25 | 31.25 9 – 10 | 14 | 10 | 1.6 | 19.6	B1 M1A1 M1 A1
	Values of D Expected Freq 4th or 5th col $\chi^2 = 13.65$	
	$\nu = 3$, $\chi_3^2(1\%) = 11.345$	B1, B1
	[Reject H_0 ,] the uniform distribution over [0, 10] is not a suitable model	A1 (9)
(b)	Area $\propto \pi R^2$ so $r = 81, -49 = \underline{\mathbf{32}}$	M1, A1
	$s = 100 - \text{"32"} - 49$ <u>or</u> $100 - 81 = \underline{\mathbf{19}}$	B1ft (3)
(c)	Not significant, Henry's model is suitable	M1, A1 (2)
(d)	H_0 : The colour\region chosen for the points is independent of gender(or no assoc') H_1 : The colour\region chosen for the points is dependent on gender(or assoc')	B1 (1)
(e)	$\frac{39 \times 65}{100}$	B1 (1)
(f)	Expected frequency for Yellow and Boys is $4.9 < 5$ so col. must be pooled/combined. [This gives a 2×3 table so $\nu = (2-1) \times (3-1) = 2$]	B1 (1)
(g)	cv = 4.605 [Not significant] so the data do <u>not</u> support Phoebe's belief oe	B1 B1 (2)
		Total 19

	Notes
(a)	2nd B1 for the correct values for D (can be implied by 40, 30, 20, and 10.) 1st M1 for at least 2 expected frequencies or clear use of a correct formula e.g. $0.4N$ 1st A1 for all the correct E_i 2nd M1 for at least 2 correct calculations from 4th or 5th column 2nd A1 for a test statistic of 13.65 (accept 13.7 to 3 sf) Awrt 13.7 only scores 2nd B1M1A1M1A1 3rd A1 for a correct conclusion rejecting the uniform model. Award provided their test statistic > 11.345
(b)	M1 for some attempt to use πR^2 to find r
(c)	M1 for a correct statement that it is not significant A1 for correctly stating that Henry's model is suitable o.e.
(d)	B1 Independence or association mentioned at least once if ditto marks used. Allow connection but not correlation.
(f)	B1 for recognising there is an $Ei < 5$ **and** need for pooling/combining oe
(g)	2ndB1 for correctly stating that Phoebe's belief is not supported by the data oe (depends on their cv being > 1.411)